

# Reduction of Squared Euclidean Norm of LMS Channel Estimator with Equalizer

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**Abstract** – In wireless communications, the channel has a typically long impulse response and the signal is highly correlated speech. The Standard Least Mean Square (LMS) channel estimator is used as a basis to this paper to estimate the active taps in channel with minimum estimation error. A Zero Tap detection scheme is used to estimate the active and inactive taps present in the unknown channel in place of Standard LMS algorithm. This scheme allows discrimination between active and inactive taps of the unknown channel. However, the detection scheme fails under colored input signal conditions. Hence, the scheme is modified to include a Tap Decoupling feature, which reverses the effect of the colored input signal. The proposed methods which are structurally consistent for both White and colored input signals. An Adaptive Equalizer which is incorporated with all proposed scheme to reduce the Squared Euclidean Norm. All the MatLab simulation results demonstrates that the design approach investigated in this paper is a promising alternative for reducing Squared Euclidean Norm of a channel estimator.

**Index Terms** – LMS Channel Estimator, Zero Tap Detection, Tap Decoupling, Equalizer, Squared Euclidean Norm.

## 1. INTRODUCTION

In this paper we present the issues and problems that are involved in various methods which are proposed. The LMS Adaptive FIR channel estimator applied via configuration shown in Figure 1 has wide range of applications particularly in Signal Processing, one such a application is echo cancellation. From the previous analysis it is evident that the LMS estimation of channels should be greatly enhanced if and only if the active taps are estimated. This leads to the proposal of a number of LMS procedures incorporating active tap detection methods.

The detection guided LMS Estimation approach requires an activity measure and activity threshold. To determine the tap to be active, the value of activity measure must be above a minimum value called activity threshold. The activity measure and activity threshold proposed here are structurally consistent for white input signals. The algorithms proposed in this paper are not based on the structural consistency and they provide asymptotic performance to the white input conditions.

In Wireless communication channel, the channel has a typically long impulse response and the signal is highly correlated speech. The correlation within the input signal causes coupling amongst the outputs of the unknown channel taps. Due to this some inactive taps may appear as active taps leads to increasing in activity noise level. Therefore such a effects leads to failure of the algorithms- [1] and [2] proposed in this paper. This algorithms [1] and [2] provides good results in the case of white input signal but failed under the colored input signals because colored input signals are highly correlated signals. Hence this algorithm is modified so that which reserves the effects of colored input signal by modifying the activity threshold. This modified algorithm is [3].

The most popular application of adaptive filters named Inverse Modeling, also known as Deconvolution has found extensive use in various engineering disciplines. The most popular application of Inverse Modeling is in communications where an inverse model, which is also called as an Equalizer, is used to reduce the channel distortion in the form of Squared Euclidean Norm . Hence we proposed a new system shown in figure 2, which is an LMS Estimator in series with Equalizer. The Equalizer proposed in this paper is constructed with Adaptive filter. This equalizer increases the asymptotic performance of an algorithms proposed in [1],[2] and [3] also decreases asymptotic error Squared Euclidean Norm than the errors estimated by the proposed system shown in figure 2.

## 2. RELATED WORK

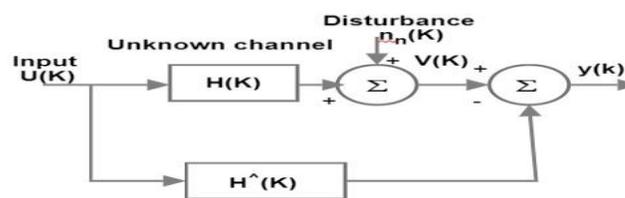


Figure 1 : LMS Channel Estimator: Existing System

Consider the system as shown in figure 1. We assumed that all signals are sampled at sampling instant  $k$ .  $U(k)$  is the

signal input to the unknown channel and the channel estimator.

Here an additive disturbance  $n_n(k)$  occurs within the Unknown channel &  $y(k)$  is the observed output from unknown channel; Therefore

$$y(k) = U(K)^T H(K) + n_n(k) \text{ where}$$

$$H(K) = [H_0, H_1, H_2, \dots, H_{n-1}]^T$$

$U(K) = [u(k), u(k-1), \dots, u(k-n+1)]^T$  our aim in this is to estimate channel parameter vector

$$\hat{H}(K) = [\hat{H}_0, \hat{H}_1, \hat{H}_2, \dots, \hat{H}_{n-1}]^T$$

Assumption 1 : The input signal is a zero mean wide sense stationary Gaussian process with a finite correlation length  $L$ :

$$\begin{aligned} R_u(j) &\triangleq E[u(k)u(k-j)] \\ &= \lim_{N \rightarrow \infty} 1/N \sum_{k=1}^N u(k)u(k-j) \\ &= \sigma_u^2, j=0 \\ &= 0, |j| > L \end{aligned}$$

$$\text{And } |R_u(j)| < \sigma_u^2, 1 \leq |j| \leq L$$

Assumption 2 : The disturbance signal is a zero mean wide sense stationary Gaussian white process which is uncorrelated with input signal:

$$\begin{aligned} R_s(j) &\triangleq E[n(k)n(k-j)] \\ &= \lim_{N \rightarrow \infty} 1/N \sum_{k=0}^{N-1} n(k)n(k-j) \\ &= \sigma_n^2, j=0 \\ &= 0, j \neq 0; \end{aligned}$$

$$\begin{aligned} Q(j) &\triangleq E[n(k)n(k-j)] \\ &= \lim_{N \rightarrow \infty} 1/N \sum_{k=0}^{N-1} n(k)u(k-j) \\ &= 0, \text{ for all values of } j; \end{aligned}$$

Assumption 3 : The time invariant n-tuple FIR modeled unknown channel,  $L(z^{-1})$ , active and correlation length of the input signal is sufficiently short such that

$$L(z^{-1}) = H_{t1} z^{-t1} + H_{t2} z^{-t2} + \dots + H_{tm} z^{-tm}$$

Where  $m < n, 0 \leq t_1 < t_2 < \dots < t_m \leq n-1$  and  $t_{i+1} - t_i > 2L, i=1, 2, \dots, m-1$ . The number  $m$  of active taps may be unknown to us.

### 2.1 Procedures

Under the assumption of a white input signal we derived the following criterion for detection of the active tap indices at  $j=t_i$

$$\begin{aligned} X_N(j) &\triangleq [\sum_{k=1}^N y(k)u(k-j)]^2 / \sum_{k=1}^N u^2(k-j) \\ &> \sum_{k=1}^N 2y^2(k) \log(N)/N \end{aligned}$$

$$\rightarrow \sum_{k=1}^N 2\sigma_y^2(k) \log(N) \text{ as } N \rightarrow \infty$$

$$\triangleq T_{\text{white}}$$

Where  $\sigma_y^2 = \text{variance of } y(k)$ .

Correlation within the input signal causes coupling amongst the outputs of the unknown channel taps and, subsequently, difficulty in determining the activity of each tap. The overabounding the effect of input signal correlation on the activity threshold. The following active tap criterion is, subsequently, proposed:

$$X_N(j) > \sum_{k=1}^N y^2(k) \log(N) / \sqrt{N}$$

$$\rightarrow \sqrt{N} \sigma_y^2 \log N \text{ as } N \rightarrow \infty$$

### 3. PORPOSED METHODS AND MODEL

In general, when the input signal is colored the measure  $X_N(j)$  provides a biased estimate of the activity of a particular (active or inactive) tap  $j$  as a result of contributions by those active taps  $i$  lying within  $L$  sample lags of  $j: |j-i| \leq L$  where  $L$  is the input signal correlation length. However, when Assumption-3 is valid, the activity measure  $X_N(j)$  provides an unbiased tap activity estimate of a subset of the  $n$  taps as follow:

$$E[X_N(j)] = 0 \text{ for } |j-t_i| > L, i=1, 2, \dots, m.$$

$$E[X_N(j)] = N \theta j^2 \sigma_u^2, \text{ for } j=t_i, i=1, 2, \dots, m$$

The first set corresponds to a subset of the inactive tap indices, the second to active tap indices. Denote the first set by  $\{S_1\}$ . The remaining inactive tap indices

$$0 < |j-t_i| \leq L, i=1 \text{ or } 2 \text{ or } \dots \text{ or } m$$

We denote by  $\{S_2\}$ . In general, the measure  $X_N(j)$  provides a biased estimate of the activity of taps within  $S_2$ . The following are the methods that are proposed in this paper. These two methods performance is compared with the Standard LMS Algorithm.

#### 3.1 Zero Tap Detection Method

Standard LMS algorithm estimate both active & inactive regions causes increasing in computational complexity, poor convergence rate. Hence Zero Tap Detection method is introduced, it estimates only active taps. To detect the 'active' taps of a time-invariant channel with the white input signals, the formula is given as:

$$C_k = \sum_{i=1}^N \frac{(v_i u_{i-k+1})^2}{\sum_{i=1}^N (u_{i-k+1})^2}$$

where  $i = \text{time index}$ ,  $k = \text{tap index}$ , and  $N$  is the number of input samples.  $C_k$  is known as the activity measure. In order to determine a tap to be active, the value of the activity measure  $C_k$  must be above a minimum value named as the activity threshold. This is given in the following formula:

$$C_k > \sum_{i=1}^N \frac{(v_i)^2 \log(i)}{i}$$

### 3.2 Tap Decoupling Method

A method named Tap Decoupling has been adapted in this paper to reduce the tap Decoupling (correlation between pulses) effect. This is done through some modifications to the activity measure. The modified activity measure in Tap Decoupling is

$$CC_k = \sum_{i=1}^N \frac{[(v_i - hv_i + H_k^{\wedge} u_{i-k+1})(u_{i-k+1})]^2}{\sum_{i=1}^N (u_{i-k+1})^2}$$

If i/p signal is white then only one tap k contribute  $c_k$ , but i/p is colored signal then many no. of taps contributes this  $c_k$

### 3.3 Proposed Channel Equalization Approach

The Proposed channel Equalization approach is shown below.

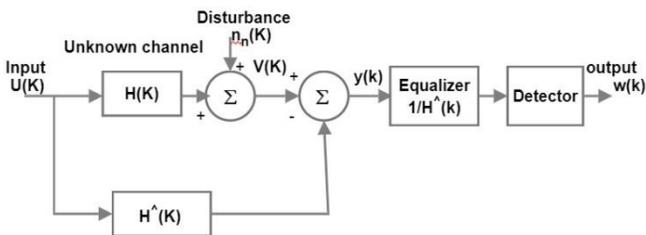


Figure 2: LMS Channel Estimator with Equalizer: Proposed System

If  $W(k)$  is equalizer Transfer function,  $H(k)$  is Transfer function of channel then the transfer function of Channel equalizer should be

$$W(k) \times H(k) = 1, \text{ where } W(k) = 1/H(k).$$

Introducing Equalizer with channel estimator increases asymptotic performance of the channel estimator. That means correct number of taps can be calculated with least estimation error.

## 4. RESULTS AND DISCUSSIONS

Simulation was done by taking unknown channel having 4-active taps with tap length of 36. Applied  $n_n(i)$  is a zero mean Gaussian variable with variance of noise = 1, Step size ( $\mu$ ) = 0.005. Zero Tap detection & Tap Decoupling were conducted based on standard LMS algorithm for following two models:

$$U(i) = W(i) \text{ for White input}$$

$$U(i) = W(i)/(1-0.8z^{-1}) \text{ for colored input signal.}$$

Where  $w(i)$  is also a white zero mean Gaussian signal of unit variance and  $z^{-1}$  is the sample delay operator. All the simulation results were shown in Table-1. The simulation was performed first without channel equalizer on LMS channel estimator with different methods which are proposed in this

paper and then with equalizer. By comparing the results we concluded that the proposed methods got successful results. These results further improved by channel equalization. The algorithms used in this paper are based on the proposed methods. They are:

- [1]. Standard LMS Algorithm: The Result of Standard LMS Algorithm with and Without Equalizer

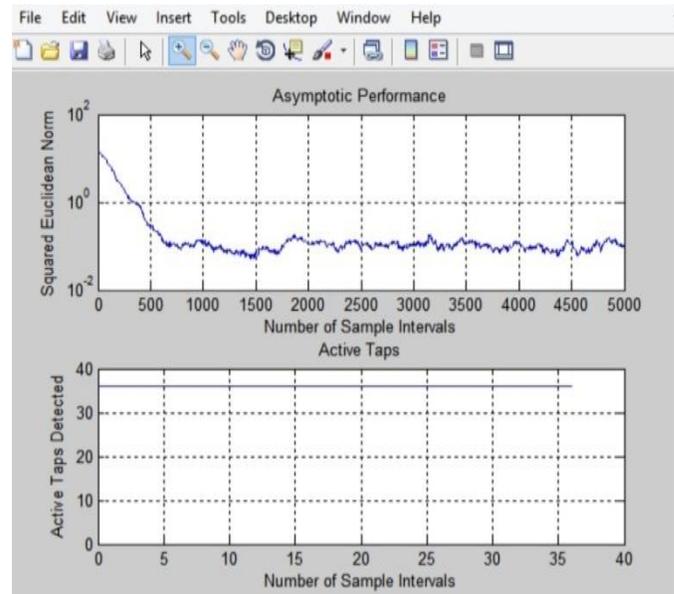


Figure 3: Standard LMS Algorithm for white input signal without Equalizer.

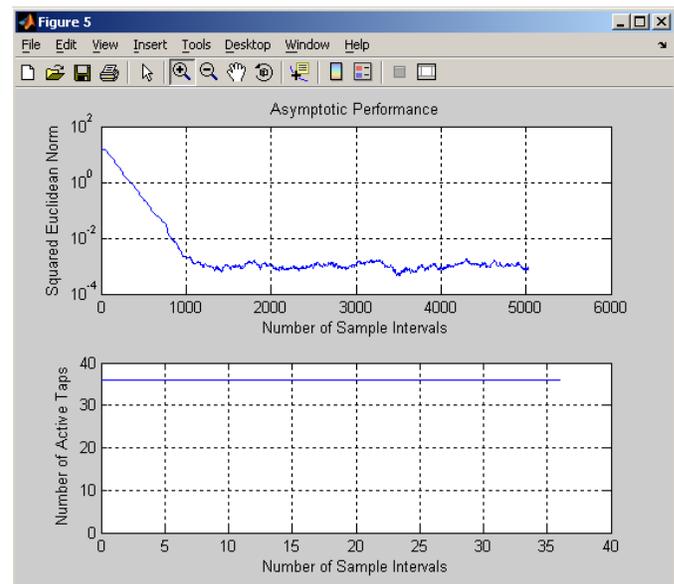


Figure 4: Standard LMS Algorithm for white input signal with Equalizer.

- [2]. Zero Tap Detection Algorithm: The Result of Zero Tap Detection with and Without Equalizer

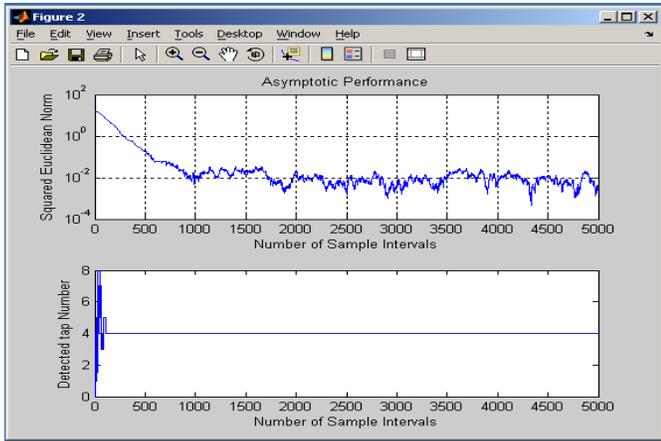


Figure 5: Zero Tap Detection Algorithm for white input signal without Equalizer.

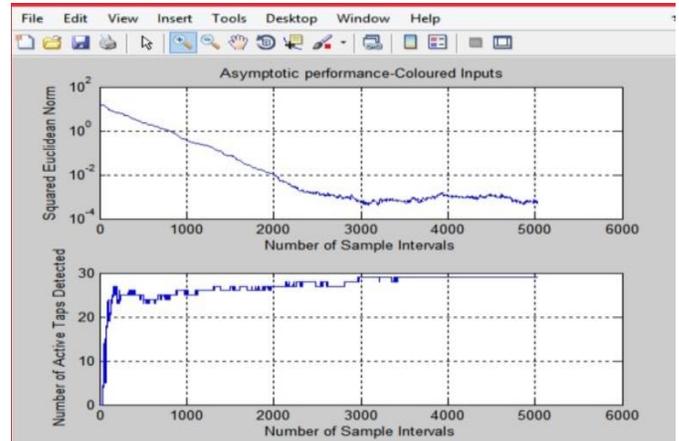


Figure 8: Zero Tap Detection Algorithm for Color input signal with Equalizer.

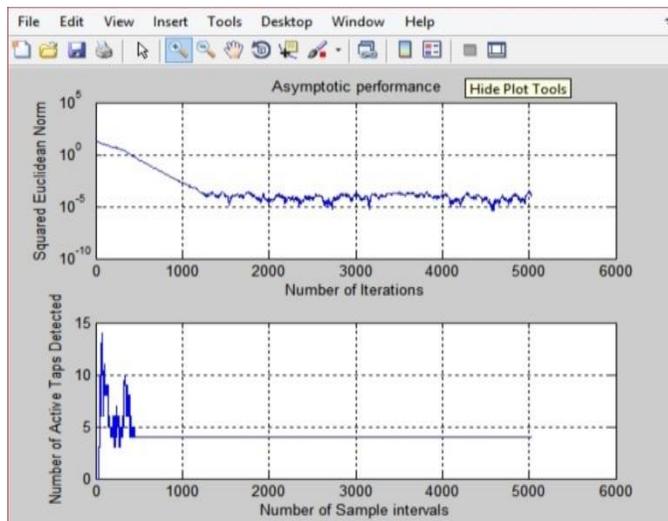


Figure 6: Zero Tap Detection Algorithm for white input signal with Equalizer.

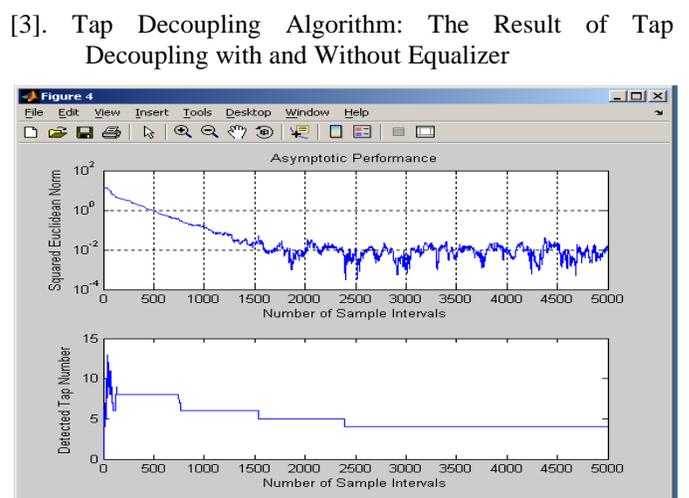


Figure 9: Tap Decoupling Algorithm for Color input signal without Equalizer.

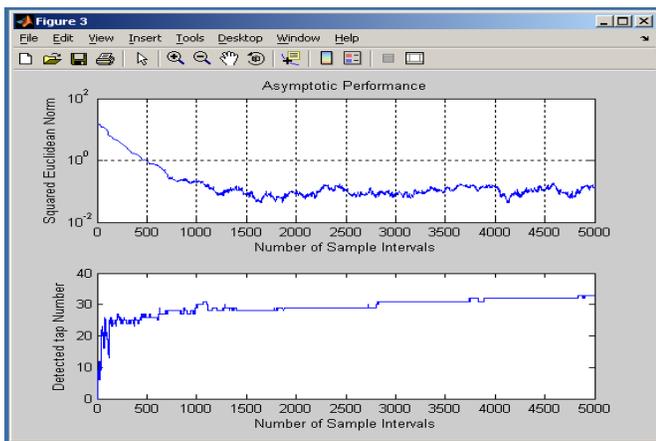


Figure 7: Zero Tap Detection Algorithm for Color input signal without Equalizer.

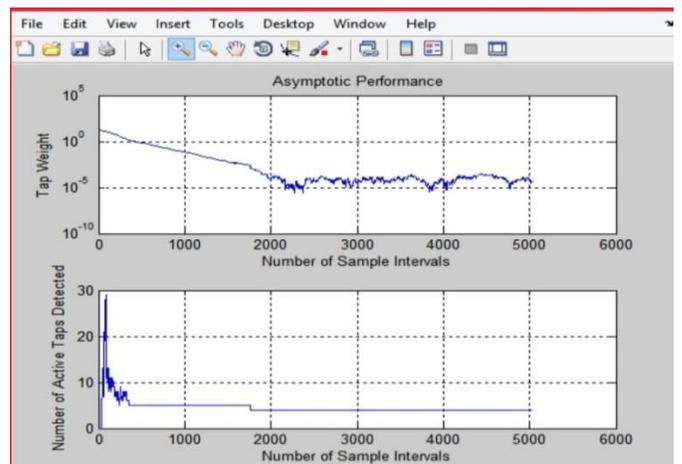


Figure 10: Tap Decoupling Algorithm for Color input signal with Equalizer.

The following Table illustrates the Squared Euclidean Norm that is Estimation Errors occurred in algorithms discussed in this paper with and without Equalizer.

Algorithm	Input Signal	No. of Active Taps Detected	Squared Euclidean Norm	
			Without Equalizer	With Equalizer
Standard LMS	White	36	$10^{-1}$	$10^{-3}$
Zero Tap Detection	White	4	$10^{-2}$	$10^{-3}$
Zero tap Detection	Color	33	$10^{-1}$	$10^{-3}$
Tap Decoupling	Color	4	$10^{-4}$	$10^{-5}$

Table1: Summary of Results

## 5. CONCLUSION

In this paper, we assume a stationary or time-invariant channel. We first implement the algorithm using Least Mean Square (LMS) for the estimation of the channel. Under white input signal conditions, we consider the use of Zero Tap Detection technique to reduce the computational burden due to the LMS estimation of 'long' channels. From the simulations conducted, it is evident that the Zero Tap Detection leads to improved asymptotic performance of the LMS estimator, but not the convergence rate. However, the detection scheme fails under colored input signal conditions, where the input signal is highly correlated speech.

The detection scheme of LMS algorithm was then modified to include the process of whitening the input signal in the initial stage of filtering. We use Tap Decoupling method, which ensures independent tap characteristics in the channel. This method allows the application of Zero Tap Detection technique to work under colored input signal conditions. Based on the simulation results produced, it is proven that the noise factor affects the asymptotic error, but not the convergence rate.

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